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Solitary Waves Solutions Of Non-Linear Schrodinger Equation With Higher Order Dispersion By Phase Amplitude Method Mathematically

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ABSTRACT: We report the complexities of nonlinear system are often characterized by some bifurcation. We can develop the simplified approximation method for the formulation of solitary wave solutions. In the past, the bifurcation and solitary waves were usually studied separately, so the underlying mechanisms were not understood well. Prediction for the solitary waves of nonlinear partial differential equations (PDEs) was possibly in but only a few of their exact or analytic solutions could be obtained directly by integration. Analytical Solution of Solitary Wave Solution of Non Linear Schrodinger Equation with higher order dispersion is shown in my paper. Furthermore, the generating mechanisms and complete results of the existence/coexistence of the solitary wave solutions, kinks, and anti-kinks, are clearer in terms of the bifurcation analysis. We study for Bright Solitary wave solution of NLSE using Phase Amplitude method and observe the nature of solutions by changing parameters. Nonlinear Schrödinger (NLS) equation, the solitary waves/solitons play an important role in both theory and applications.

Keywords: Non Linear Schrodinger Equation, Solitary Waves.

1. INTRODUCTION

There are number of solutions for discovering many simplified equations for Nonlinear Schrodinger effects in order to analyze solitary wave solutions. Some of these models admit exact solutions which play fundamental role for theory of the related equations and have large application values. One of the fundamental objects of nature and increasing interest of researchers in propagation of nonlinear waves in dynamical system solitary wave solution is one important parameter. Also, in various scientific and engineering fields Nonlinear Schrodinger plays a great role for development of solitary wave solutions. A Travelling Wave Solutions of Fractional Order Coupled Burgers? Equations by (*G'G*) -Expansion Method [5], also, He, J.H. (1999) researched for Some Applications of Nonlinear Fractional Differential Equations[4] and Johnson, R.S. (1970) discovered A Non-Linear Equation Incorporating Damping and Dispersion[1]. The solitary wave ansatz method [13] [14] is rather heuristic and processes significant features that make it practical for the determination of single solitons solutions for a Wide class of nonlinear evolution equations. The solitary wave and shock wave solitons have been obtained, using solitary wave ansatz method

In literature, numerous investigations on nonlinear effects are reported, in order to analyze and suppress the nonlinear effects. In recent years, many methods had been obtained in succession in order to study the exact solitary wave solutions of mathematical-physical equation, such as reverse-scattering method, Backlund transformation, the wadati trace method, Hirota bilinear forms, the sine-cosine method and first integral method. Since the discovery of soliton by Kruskal and Zabusky [1] there have been many significant theoretical and numerical contributions to the development of the solitons theory [2], [3], [4], [5]. There are various issues of NLSEs that need to be addressed. These include the integrability aspect, conservation laws, wave interactions and many more. Including various aspects of computation for conserved quantities, wave-wave interaction, quasi-stationary solutions one can obtain number of solutions for NLSEs. Hence, the very first steps constitute the prime importance of problem and then compute its solution.

2. GOVERNING EQUATIONS

One of these equations is the generalized normalized Non-Linear Schrodinger equation is written as follows:

$$\frac{\partial q}{\partial \xi} = -i\frac{a_2}{2}\frac{\partial^2 q}{\partial \tau^2} + \frac{a_3}{6}\frac{\partial^3 q}{\partial \tau^3} + i\frac{a_4}{24}\frac{\partial^4 q}{\partial \tau^4} + iq|q|^2$$

Here,

$q(\xi,\tau)$ represents the normalized slowly varying envelope equation of the electric field and ξ *and* τ are respectively the normalized distance along the fibre and time with the frame of the reference moving along the fibre at group velocity respectively,

$a_2 = \text{sgn}(\beta_2), a_3 = -\frac{\beta_3}{|\beta_2|}T_0$ and $a_4 = \frac{\beta_4}{|\beta_2|}T_0{}^2$ and

here β_2, β_3 and β_4 are the so called group velocity dispersion, the third order dispersion and the fourth order dispersion parameter. T_0 is the initial pulse width. In order to solve equation (1), we take $q(\xi,\tau) = u(\xi,\tau)e^{i\phi(\xi,\tau)}$ where $u(\xi,\tau)$ the complex envelope function is and $\phi(\xi,\tau) = k\xi - \Omega\tau$ is the linear phase shift function with k *and* Ω are the normalized wave vector and frequency respectively.

Since $\phi(\xi,\tau) = k\xi - \Omega\tau$ hence $q(\xi,\tau) = u(\xi,\tau)e^{i(k\xi-\Omega\tau)}$

Therefore, $$\frac{\partial q}{\partial \xi} = e^{i(k\xi-\Omega\tau)}\left[\frac{\partial u}{\partial \xi} + iuk\right] \quad (2)$$

$$\frac{\partial^2 q}{\partial \tau^2} = e^{i(k\xi-\Omega\tau)}\left[\frac{\partial^2 u}{\partial \tau^2} - 2i\Omega\frac{\partial u}{\partial \tau} - \Omega^2 u\right] \quad (3)$$

$$\frac{\partial^3 q}{\partial \tau^3} = e^{i(k\xi-\Omega\tau)}\left[\frac{\partial^3 u}{\partial \tau^3} - 3i\Omega\frac{\partial^2 u}{\partial \tau^2} - 3\Omega^2\frac{\partial u}{\partial \tau} + i\Omega^3 u\right] \quad (4)$$

$$\frac{\partial^4 q}{\partial \tau^4} = e^{i(k\xi-\Omega\tau)}\left[\frac{\partial^4 u}{\partial \tau^4} - 4i\Omega\frac{\partial^3 u}{\partial \tau^3} - 6\Omega^2\frac{\partial^2 u}{\partial \tau^2} + 4i\Omega^3\frac{\partial u}{\partial \tau} + \Omega^4 u\right] \quad (5)$$

On Substituting the Equations from (2) to (5) in Equation (1) We Obtained

$$\frac{\partial u}{\partial \xi} + iku = -i\frac{a_2}{2}\frac{\partial^2 u}{\partial \tau^2} - a_2\Omega\frac{\partial u}{\partial \tau} + i\frac{a_2}{2}\Omega^2 u + \frac{a_3}{6}\frac{\partial^3 u}{\partial \tau^3}$$
$$-i\frac{a_3}{2}\Omega\frac{\partial^2 u}{\partial \tau^2} - \frac{a_3}{2}\Omega^2\frac{\partial u}{\partial \tau} + i\frac{a_3}{6}\Omega^3 u + i\frac{a_4}{24}\frac{\partial^4 u}{\partial \tau^4}$$
$$-\frac{a_4}{6}\Omega\frac{\partial^3 u}{\partial \tau^3} - i\frac{a_4}{4}\Omega^2\frac{\partial^2 u}{\partial \tau^2} - \frac{a_4}{6}\Omega^3\frac{\partial u}{\partial \tau}$$
$$+i\frac{a_4}{24}\Omega^4 u + iu|u|^2$$

We get;

$$\frac{\partial u}{\partial \xi} + b_1\frac{\partial u}{\partial \tau} + ib_2\frac{\partial^2 u}{\partial \tau^2} - b_3\frac{\partial^3 u}{\partial \tau^3} - ib_4\frac{\partial^4 u}{\partial \tau^4} + ib_5 u - iu|u|^2 = 0 \quad (6)$$

Where,

$$b_1 = a_2\Omega + \frac{a_3}{2}\Omega^2 + \frac{a_4}{6}\Omega^3 \; ; \; b_2 = \frac{1}{2}(a_2 + a_3\Omega + a_4\Omega^2) \; ;$$

$$b_3 = \frac{1}{6}(a_3 + a_4\Omega) \; ; \; b_4 = \frac{a_4}{24}$$

And $b_5 = k - (\frac{a_2}{2}\Omega^2 + \frac{a_3}{6}\Omega^3 + \frac{a_4}{24}\Omega^4)$

2.1 Solitary Wave Solutions

Further for Bright Soliton, we take the function as $u(\xi,\tau) = BSech\left[W\left(\tau - \frac{\xi}{V}\right)\right]$ Where B, W and V are the amplitude, the pulse width and Velocity of Soliton in normalized units.

Since $u(\xi,\tau) = BSech\left[W\left(\tau - \frac{\xi}{V}\right)\right]$ and Taking $\left[W\left(\tau - \frac{\xi}{V}\right)\right] = \chi$

$$\frac{\partial u}{\partial \xi} = \left(\frac{BW}{V}\right) Sech\left[W\left(\tau - \frac{\xi}{V}\right)\right] Tanh\left[W\left(\tau - \frac{\xi}{V}\right)\right]$$

Implies $$\frac{\partial u}{\partial \xi} = \left(\frac{BW}{V}\right) Sech\chi\, Tanh\chi \quad (7)$$

Also, $$\frac{\partial u}{\partial \tau} = -BW\; Sech\left[W\left(\tau - \frac{\xi}{V}\right)\right] Tanh\left[W\left(\tau - \frac{\xi}{V}\right)\right]$$

Hence, $$\frac{\partial u}{\partial \tau} = -BW\; Sech\chi\, Tanh\chi \quad (8)$$

$$\frac{\partial^2 u}{\partial \tau^2} = -BW^2\left(Sec^3h\left[W\left(\tau - \frac{\xi}{V}\right)\right]\right) - Tan^2h\left[W\left(\tau - \frac{\xi}{V}\right)\right] Sech\left[W\left(\tau - \frac{\xi}{V}\right)\right]$$

Hence, $$\frac{\partial^2 u}{\partial \tau^2} = -BW^2\left(Sec^3h\chi - Tan^2h\chi\; Sech\chi\right)$$

$$\frac{\partial^2 u}{\partial \tau^2} = -BW^2\left(2Sec^3h\chi - Sech\chi\right) \quad (9)$$

And $$\frac{\partial^3 u}{\partial \tau^3} = -BW^3\left(-6Sec^3h\chi\, Tanh\chi + Sech\chi Tanh\chi\right) \quad (10)$$

$$\frac{\partial^4 u}{\partial \tau^4} = -BW^4\left(-24Sec^5h\chi + 20Sec^3h\chi - Sech\chi\right) \quad (11)$$

Putting Equations (7) to (11) in Equation (6) We Get;

$$\left(\frac{BW}{V}\right) Sech\chi\, Tanh\chi + ib_1(-BW\; Sech\chi\, Tanh\chi) +$$
$$ib_2\left(-BW^2\left(2Sec^3h\chi - Sech\chi\right)\right)$$
$$+ib_3 BW^3\left(-6Sec^3h\chi\, Tanh\chi + Sech\chi Tanh\chi\right) +$$
$$ib_4 BW^4\left(-24Sec^5h\chi + 20Sec^3h\chi - Sech\chi\right) +$$

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$$ib_5 BSech\chi - iBSec^3h\chi = 0$$

Separating the Real and Imaginary Part We Obtained

$$\left(\frac{BW}{V} - b_1 BW + b_3 BW^3\right)Sech\chi\, Tanh\chi + \left(6b_3 BW^3\right)Sec^3h\chi\, Tanh\chi = 0 \tag{12}$$

$$\left(b_2BW^2 - b_4BW^4 + b_5B\right)Sech\chi + \left(-2b_2BW^2 + 20b_4BW^4 - B^3\right)Sec^3h\chi - \left(24b_4BW^4\right)Sec^5h\chi = 0 \tag{13}$$

We Can Equate the Coefficients of Linearly Independent Terms to be Zero and We Will Obtained Four Independent equations. By Getting These Four Equations and Introducing Some Constraints We Can Get All Parameters of Solitary Waves as Follows

$$\Omega = -\frac{a_3}{a_4},\quad V = \left(b_1 - b_3W^2\right)^{-1}$$

$$, k = b_4 W^4 - b_2 W^2 + \frac{a_2}{2}\Omega^2 + \frac{a_3}{6}\Omega^3 + \frac{a_4}{24}\Omega^4 \text{ and}$$

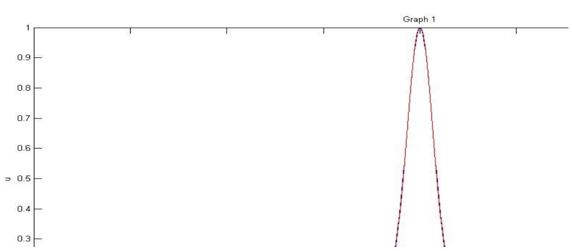
$$W^2 = \frac{b_2}{10 b_4}.$$

In Order to understand the nature of the Solution We Choose following Parameters;

$$\beta_2 = -50\, ps^2 / Km,\quad \beta_3 = 0.5\, ps^4 / Km \text{ and}$$
$$\gamma_1 = 2736\, W^{-1} Km^{-1}, \gamma_2 = 2.63\, W^{-2}\, Km^{-1}, T_0 = 10\, fs$$
$$\text{and } P_0 = 500.$$

3. RESULTS AND DISCUSSIONS

Graph 1: Results in graph 1 shows peak value propagation at

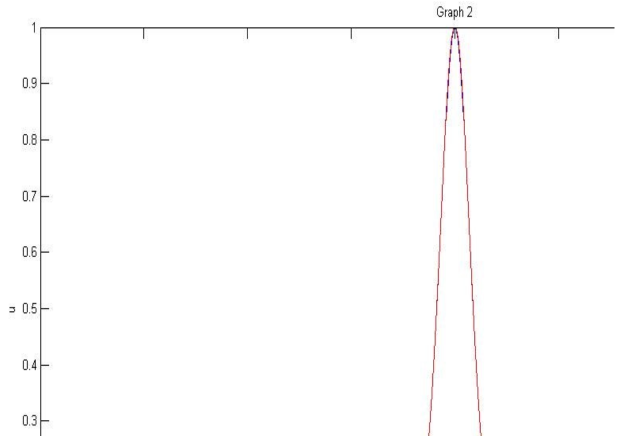


different parameters where To=100, 50, 10 and further can be formulated into more different parameters. To=100, 50, 10 here gives the best results for our research.

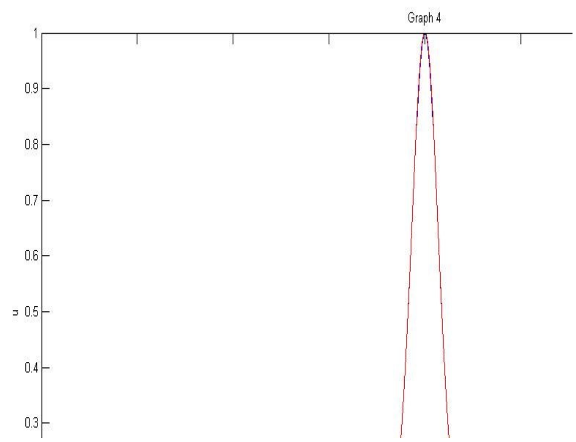
Graph 2: For our second result we have changed the parameter to To=1 and have defined a better performance analysis.

Graph 3:

Graph 4: In this result, we have shown steeping effect

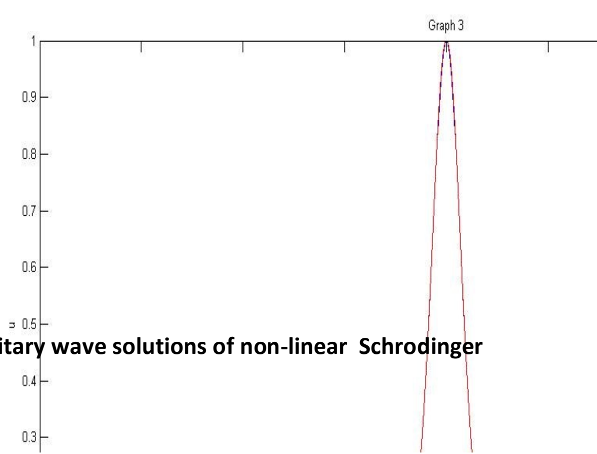


including higher order dispersion at To=100 which is defined parameter and has been configured using ansatz method for much better performance from previous results.



4. CONCLUSION

In this work, we have used the higher order nonlinear Schrodinger equation with cubic-quintic terms along with the self-steeping effect and higher order dispersion effects to describe the propagation of ultra-short fem to second pulses. We have observed different modulations for different parameters and we have obtained the bright solitary wave solution by the phase amplitude ansatz method mathematically.



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